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ELECTRICAL RESONANCE ANALYSIS ON RLC SERIES CIRCUIT

FLORIN GABRIEL POPESCU[1], TITU NICULESCU[2], DRAGOS PASCULESCU[3], RAZVAN SLUSARIUC[4], TEODORA LAZAR[5], FLORIN MURESAN-GRECU[6]

Abstract: In a series RLC circuit, resonance occurs when the inductive reactance (XL) and capacitive reactance (XC) are equal in magnitude but opposite in phase.

It's important to note that in a series RLC circuit, the total impedance is the vector sum of the resistance (R), inductive reactance (XL), and capacitive reactance (XC). The resonant frequency is the frequency at which XL and XC are equal, resulting in the cancellation of these two reactances, leaving only the resistance.

In summary, the resonant frequency in a series RLC circuit is the frequency at which the inductive and capacitive reactances cancel each other out, leading to a minimum impedance and maximum current flow.

Key words: circuit, current, frequency, power, resonance, voltage.

1. INTRODUCTION

Alternating current electrical circuits are those circuits supplied with alternating electromotive voltages, usually sinusoidal in time. These circuits are of particular importance in engineering because of their many advantages in the generation, transmission, distribution and use of electricity [1], [3], [5].

In a series RLC circuit a frequency point occurs when the inductive reactance of the inductor becomes equal in value to the capacitive reactance of the capacitor. In other words, $X_L = X_C$. The point at which this occurs is called the **resonant frequency** point of the circuit, and as we analyze a series RLC circuit, this resonant frequency produces a **series resonance** [4], [7], [9].

[1] *Ph.D., Associate Prof. Eng.,University of Petroșani, floringabriel82@yahoo.com*
[2] *Ph.D., Associate Prof. Eng., University of Petroșani, niculescu_titu@yahoo.com*
[3] *Ph.D., Associate Prof. Eng., University of Petroșani, pdragos_74@yahoo.com*
[4] *Ph.D., Lecturer Eng.,University of Petroșani, razvansir@yahoo.com*
[5] *Ph.D., Lecturer Eng., University of Petroșani, teomititica@yahoo.com*
[6] *Ph.D., Student Eng., University of Petroșani, flomavon2002@yahoo.com*

Series resonant circuits are some of the most important circuits used in electrical and electronic circuits [22]. They can be found in various forms such as AC mains filters, noise filters and also in radio and television circuits producing a highly selective tuning circuit for the reception of different frequency channels [2], [11], [17].

Resonance is a very valuable property of AC reactive circuits used in a variety of applications.

2. CASE STUDY

It is well known that passive AC electrical circuits of the Resistor-Coil-Capacitor (RLC) type achieve resonance (and therefore dissipate maximum energy on the resistor in the form of heat) at the resonant frequency, where the electrical impedance is minimum [8], [16], [18].

An experiment organized on an experimental stand formally described in Fig.1 can confirm this finding.

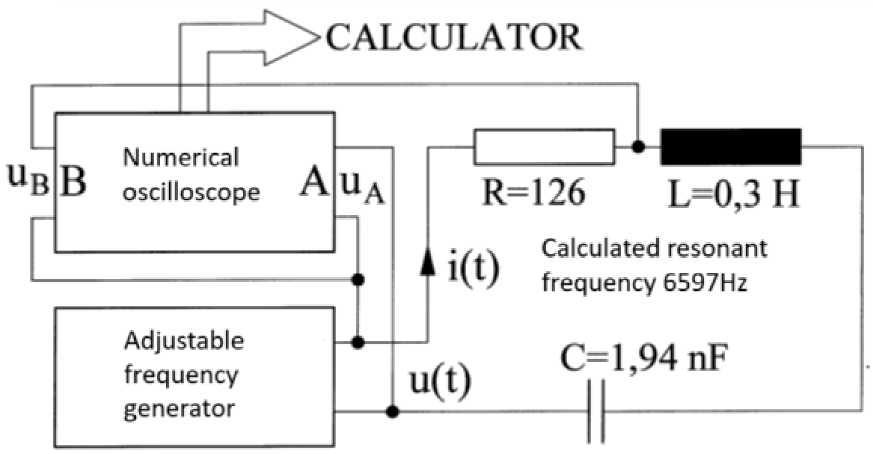

Fig.1. Schematic for electrical resonance analysis on the RLC series circuit

A frequency-tunable harmonic signal generator feeds a series circuit consisting of a resistor with resistance $R = 126\ \Omega$, a coil with inductance $L = 0.3\ H$ and a capacitor with capacitance $C = 1.94\ nF$. The resonance of this circuit is at pulse $\omega = 2\cdot\pi\cdot f$ (here f is the frequency) for which the electrical impedance modulus Z is minimum:

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + \left(\omega \cdot L - \frac{1}{\omega \cdot C}\right)^2} \qquad (0.1)$$

In this case the moduli of the inductive X_L and capacitive X_C reactances are equal. This condition is achieved for the excitation pulse $\omega = \frac{1}{\sqrt{L \cdot C}}$, i.e. the frequency: $f = \frac{1}{2 \cdot \pi \cdot \sqrt{LC}}$.

One can further investigate the evolution of the active power absorbed by this circuit for different values of the excitation frequency (at and near resonance) [6], [10], [23].

The active electrical power absorbed by the RLC circuit from the signal generator can be described with the relation:

$$P = \frac{1}{T} \cdot \int_{t=0}^{T} u(t) \cdot i(t) \cdot dt \qquad (0.2)$$

The active electrical power appears as the average value of the instantaneous power *p(t) = u(t)·i(t)* (product of instantaneous voltage - instantaneous current) calculated by integrating over a period *T=1/f* the instantaneous excitation voltage *u(t).* Instantaneous voltage and instantaneous current can be described as harmonic quantities with the relations [13], [19]:

$$\begin{aligned} u(t) &= U\sqrt{2}\sin(\omega t) \\ i(t) &= I\sqrt{2}\sin(\omega t - \varphi) \end{aligned} \qquad (0.3)$$

Here *U* and *I* are called rms values of voltage and current respectively (sometimes also called *rms* values, abbreviation of *root mean square*), the products $U\sqrt{2}$ and $I\sqrt{2}$ are the amplitudes of the harmonic developments, φ is the phase shift between current and voltage which can be positive or negative, depending on the character of the reactance, inductive ($X_L > X_C$) or capacitive ($X_C > X_L$) [12], [15], [20].

With relations (1.3) the evolution of the active power in relation (1.2) can be written:

$$P = \frac{2}{T} \cdot \int_{t=0}^{T} U\sin(\omega t) \cdot I\sin(\omega t - \varphi) \cdot dt \qquad (0.4)$$

respectively

$$P = \frac{2 \cdot U \cdot I}{T} \cdot \int_{t=0}^{T} \sin(\omega t) \cdot \left[\sin(\omega t) \cdot \cos(\varphi) - \cos(\omega t) \cdot \sin(\varphi) \right] \cdot dt =$$
$$= 2 \cdot U \cdot I \cdot \cos(\varphi) \cdot \frac{1}{T} \int_{t=0}^{T} \sin(\omega t) \cdot \sin(\omega t) \cdot dt -$$
$$-2 \cdot U \cdot I \cdot \sin(\varphi) \cdot \frac{1}{T} \int_{t=0}^{T} \sin(\omega t) \cdot \cos(\omega t) \cdot dt$$

I mean:

$$P = 2 \cdot U \cdot I \cdot \cos(\varphi) \cdot F_1 - 2 \cdot U \cdot I \cdot \sin(\varphi) \cdot F_2 \qquad (0.5)$$

With:

$$F_1 = \frac{1}{T} \int_{t=0}^{T} \sin(\omega t) \cdot \sin(\omega t) \cdot dt$$
$$F_2 = \frac{1}{T} \int_{t=0}^{T} \sin(\omega t) \cdot \cos(\omega t) \cdot dt \qquad (0.6)$$

Although there are mathematical reasons on the basis of which the factors F_1 and F_2 can be calculated, a simple numerical calculation method is presented here [14], [21]. Assuming a discretization of the expression of the time variable of the form: $t \approx k \cdot \Delta t$, one can consider $\Delta t \approx dt$, one can write $T \approx n \cdot \Delta t$, and the factor F_1 can be given as:

$$F_1 \approx \frac{1}{n \cdot \Delta t} \sum_{t=0}^{t=n \cdot \Delta t} \sin(\omega t) \cdot \sin(\omega t) \cdot \Delta t = \frac{1}{n} \sum_{t=0}^{t=n \cdot \Delta t} \sin(\omega t) \cdot \sin(\omega t) \qquad (0.7)$$

Plot the evolution of the function *sin(ωt)·sin(ωt)* over a period *T = 1s* and calculate according to (1.7) the value of F_1 as the arithmetic mean (of *n* values) of this function over the period *T* .

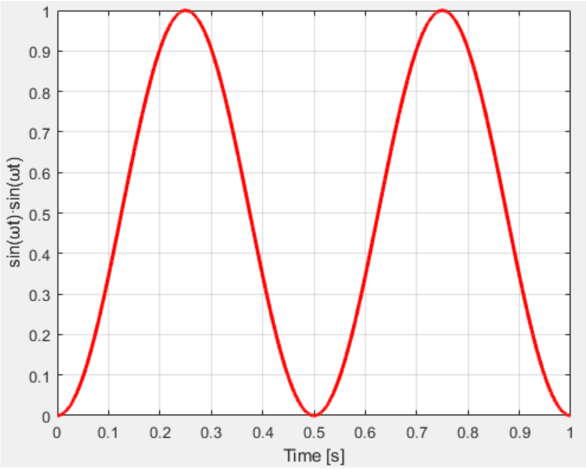

Fig.2. Graphical representation of the function sin(ω·t)· sin(ω·t) on an interval equal to period T=1 s. F1=1/2

According to the representation in Fig.2, it is first observed that the *sin(ωt)·sin(ωt)* function has double the frequency of the instantaneous current and voltage. It is also observed that the mean value of this function is 1/2, so $F_1 = \frac{1}{2}$.

Absolutely identical considerations allow writing the factor F_2 according to:

$$F_2 \approx \frac{1}{n \cdot \Delta t} \sum_{t=0}^{t=n\cdot\Delta t} \sin(\omega t) \cdot \cos(\omega t) \cdot \Delta t = \frac{1}{n} \sum_{t=0}^{t=n\cdot\Delta t} \sin(\omega t) \cdot \cos(\omega t) \qquad (0.8)$$

The evolution of the function *sin(ωt)·cos(ωt)* over a period *T = 1s* will also be plotted.

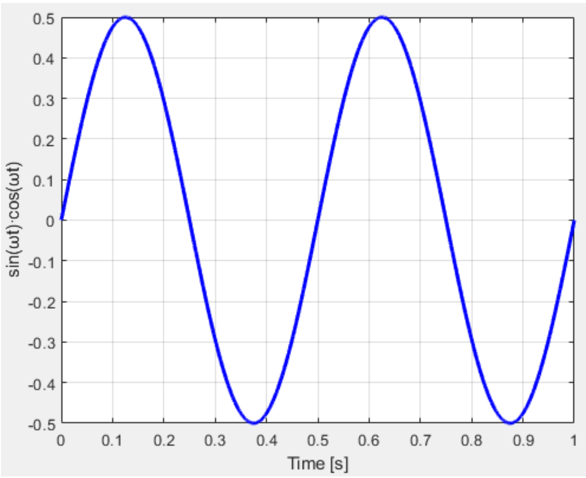

Fig.3. Graphical representation of the function sin(ω·t)· cos(ω·t) on an interval equal to period T=1 s. F2 = 0

In this case, according to the representation in Fig.3, it is also observed that the function *sin(ω·t)· cos(ω·t)* has double the frequency of the instantaneous voltage and a zero value of the factor F_2 is obtained.

With $F_1 = \frac{1}{2}$ and $F_2 = 0$, the relation (1.5) becomes:

$$P = U \cdot I \cdot \cos\left(\varphi\right) \tag{0.9}$$

The active power is in fact the product of the rms values of the instantaneous voltage and current and the so-called power factor *cos(φ)*.

Let's take this approach again from relation (1.2). According to Fig.1, using a digital oscilloscope, two voltages are sampled and transmitted to a computer: u_A and u_B. The two voltages describe in this order the instantaneous voltage *u(t)* and the instantaneous current *i(t)* (deduced on the basis of the relation $i(t) = u_B(t)/R$ applying Ohm's law) applied to the RLC circuit, and used in relation (1.2). Based on these considerations relation (1.2) can be rewritten as:

$$P = \frac{1}{T \cdot R} \cdot \int_{t=0}^{t=T} u_A\left(t\right) \cdot u_B\left(t\right) \cdot dt \tag{0.10}$$

The voltage values taken by the computer via the oscilloscope are sampled and converted into numerical format. Let Δt be the sampling interval of the two voltages (considered sufficiently small). Let the period T be expressed as: $T \approx n \cdot \Delta t$, with n an integer (as number of samples). Let the numerical expression of the voltages involved in (1.10) be of the form: $u_A(t) = u_A(k \cdot \Delta t)$, respectively $u_B(t) = u_B(k \cdot \Delta t)$, assuming a discrete (sampled) time representation of the form $t = k \cdot \Delta t$.

To perform the calculations, the relation (1.10) is approximated by finite summation, (assuming that $dt \approx \Delta t$, for sufficiently small Δt) according to:

$$P \approx \frac{1}{n \cdot \Delta t} \cdot \frac{1}{R} \sum_{k=1}^{n} \left[u_A\left(k \cdot \Delta t\right) \cdot u_B\left(k \cdot \Delta t\right) \cdot \Delta t \right] \tag{0.11}$$

In relation (1.11) the constant Δt is removed from the sum and simplified so that it becomes:

$$P \approx \frac{1}{n} \cdot \frac{1}{R} \sum_{k=1}^{n} \left[u_A\left(k \cdot \Delta t\right) \cdot u_B\left(k \cdot \Delta t\right) \right] \tag{0.12}$$

Based on relation (1.12), the power absorbed by the RLC series circuit can be measured. Let there be three distinct situations of supplying the RLC circuit with voltage u_A *(t)* of constant amplitude but for three different values of frequency in the resonance region.

Fig.4 shows the evolution of the voltages u_A and u_B for a supply signal frequency of 6122 Hz (sub-resonant).
For a sampling interval of $\Delta t = 100\ ns$, applying the formula (1.12) -where $n = 1633$- leads to the determination of the active electrical power $P = 50.3\ \mu W$.

Also marked on the figure is the current-voltage phase shift, $\varphi = 80.50$ (the current is phase-shifted following the voltage, according to (1.3) it later passes through zero).

If the phase shift is positive, the reactance of the circuit is said to be inductive. The active electrical power absorbed is very low, mainly because of the very low power factor. According to (1.9), $cos(\varphi) = 0.165$.

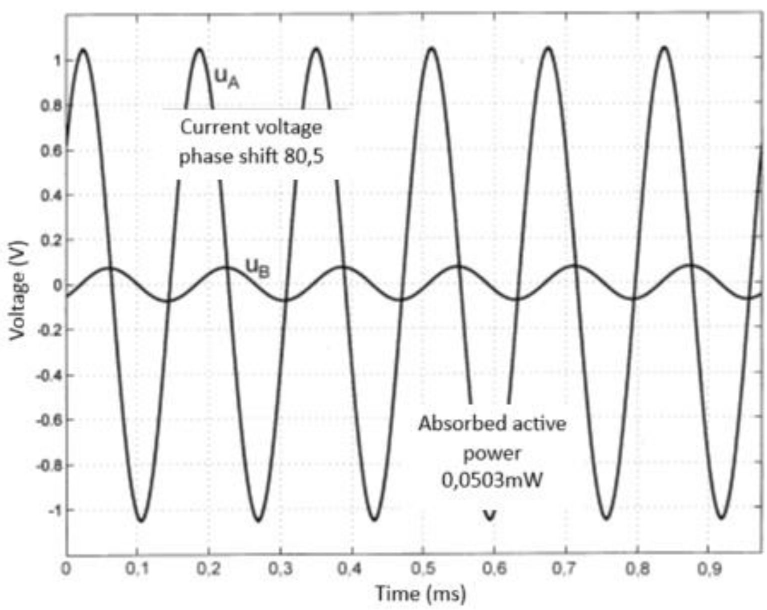

Fig.4. Relationship between the voltages u_A and u_B for a sub-resonant frequency (6122 Hz) supplying the circuit; P = 50.3 μW

Fig.4 shows the evolution of the voltages u_A and u_B for a supply signal frequency of 6364 Hz (near resonance).

Two aspects are worth noting here in comparison with Fig.4. The first is the increase in the amplitude of the voltage u_B , hence the current in the circuit. The second is related to the drastic reduction of the current-voltage phase shift to the value $\varphi = 11.91^o$ (the reactance of the circuit is still inductive).

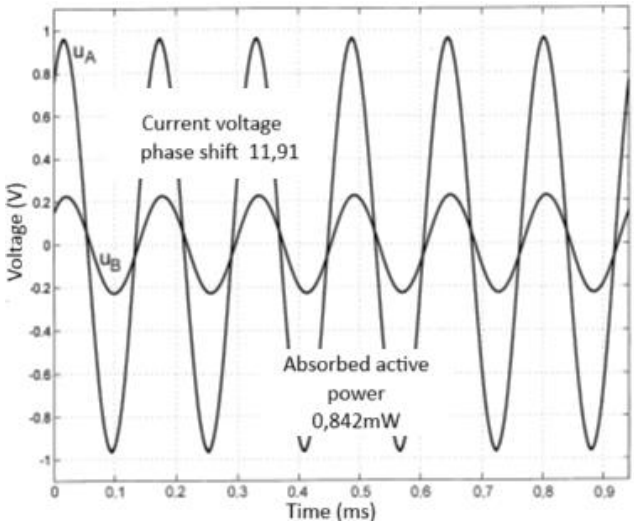

Fig.5. Relationship between voltages u_A and u_B for a circuit supply frequency near resonance (6364 Hz); P = 842 μW

Both aspects contribute to the increase of the absorbed active power which here has the value $P = 842\ \mu W$. It is obvious that when the circuit is excited at resonance the current-voltage phase shift will be zero, the power factor is maximum $cos(\varphi) = 1$, *the* reactance is zero ($X_L = X_C$), the character of the load is purely resistive, the effects induced in the circuit by the coil and capacitor disappear. Obviously, the active power absorbed by the circuit will be maximum. Practically here resonance is difficult to achieve because of the difficulties of frequency tuning.

Fig.6 shows the evolution of the voltages u_A and u_B for a supply signal frequency of 7212 Hz (over-resonant).

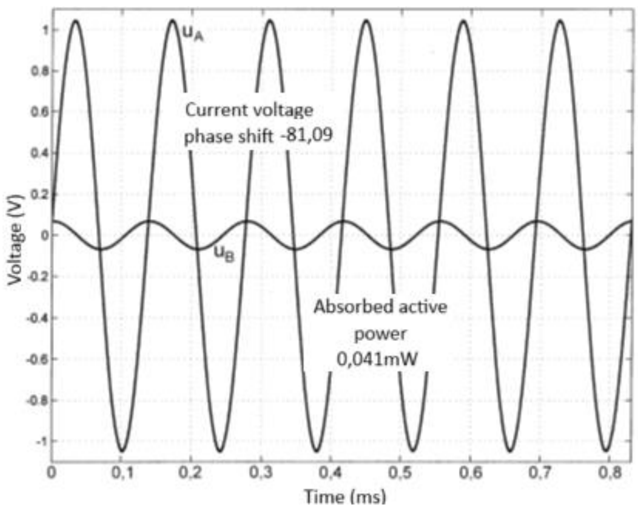

Fig.6. Relationship between the voltages u_A and u_B for an over-seasonal supply voltage frequency (7212 Hz). P = 41 μW

Two points are worth noting here. The value of the voltage u_B (hence the current in the circuit) drops again, apparently similar to the value in figure 3 and the current-voltage phase shift changes radically, this time $\varphi = -81.09^o$ is negative which means a purely capacitive character of the inductive reactance ($X_C > X_L$), the current passes through null values before the voltage. Both aspects contribute to the definition of a low value of absorbed active power: $P = 41\ \mu W$.

3. CONCLUSIONS

Simulink comes with a comprehensive set of pre-built blocks that represent various electrical components and mathematical operations. This includes blocks specific to RLC circuits, such as resistors, inductors, capacitors, and more. These pre-built blocks simplify the process of building complex circuit models.

From the three examples described in Fig.4, 5 and 6, an important conclusion is that the active power absorbed by the RLC series circuit is maximum at resonance and tends asymptotically to zero for sub- and super-resonant excitation frequency values. In all three cases the absorbed active power is dissipated as heat by the resistor due to the Joule effect.

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